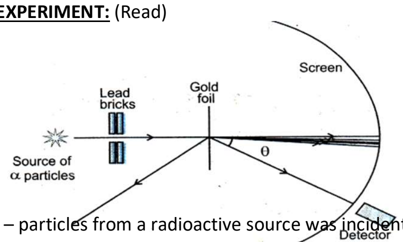
GEIGER – MARSEN EXPERIMENT: (Read)

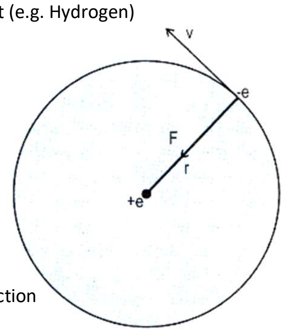


A narrow beam of α – particles from a radioactive source was incident on a gold foil. The scattering was detected by the detector fixed on a rotating stand. Detector has a Zinc Sulphide screen and microscope. The whole setup is enclosed in an evacuated chamber. The number of α particles and their scattering angles (θ) are observed.
The observation showed most of the alpha particles passed undeviated. About 0.14% scattered more than 1^{o}. Few got deflected slightly. Very few (1 in 8000) deflected by more than 90^{o}. Some particles even bounced back with 180^{o}.

RUTHERFORD'S MODEL of an Atom:
>>The atom has tiny positively charged core called nucleus
>> The total positive charge and the entire mass (99.9%) of the atom is confined in nucleus
>> The nucleus is surrounded by negatively charged electrons, orbiting round the nucleus in circular orbits
>>As an atom is electrically neutral, the positive charge on nucleus is equal to the total negative charge of all the orbiting electrons
>> As the size of the nucleus (10^{-15}m), about 100000 times smaller than the size of atom. Thus atom mostly consists of empty space.

Failures:
>>The stability of atomic structure and hydrogen line spectrum could not be explained on the basis of his model.
>> Accelerated charge radiates energy. Hence, the energy of accelerated electron should continuously decrease and follow inward spiral path and finally fall into the nucleus. Thus no stable atom could exist.
>> If the electron spirals inwards, their angular velocity and hence frequency would increase continuously. Thus, they would emit energy with continuously increasing frequency (decreasing wavelength) and hence we would get a continuous spectra. In which case no atom would have a line spectra, which we know is incorrect (e.g. Hydrogen)

BOHR's postulates or BOHR'S MODEL:
Postulate 1: The electron in a hydrogen atom revolves in circular orbit around the nucleus with nucleus at the centre of orbit. The necessary centripetal force for circular motion is provided by electrostatic force of attraction between the positively charged nucleus and negatively charged electron.



Centripetal force = Electrostatic Force of Attraction
$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$$

Postulate 2: The electron revolves around the nucleus only in those orbits for which the angular momentum is equal to an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant. These orbits are called stable or stationary (or permitted or quantized or Bohr orbits) and electron does not radiate energy while revolving in these orbits.
Angular Momentum = mvr = n $\frac{h}{2\pi}$, where n = 1,2,3,4,….
n is called the principle quantum number.

Postulate 3: When electron jumps from orbit of higher energy to an orbit of lower energy, it radiates energy in the form of quanta or photons. The energy of emitted photon is equal to the difference between energies of two orbits in which transition is taking place.

Suppose an electron from n^{th} higher orbit, jumps to p^{th} lower orbit. Let E_n and E_p be the energies of electron in n^{th} and p^{th} orbit respectively, then
Energy radiated = $E_n - E_p$ = hv

Radius of Bohr's orbit:
Consider an electron revolving around the nucleus in circular orbit of radius r. According to Bohr's first postulate
Centripetal force = electrostatic force of attraction
$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} \qquad thus\ v^2 = \frac{e^2}{4\pi\varepsilon_o mr} \ldots\ldots\ldots\ldots..(I)$$
$$According\ to\ Bohr's\ 2nd\ postulate$$
$$Angular\ momentum\ mvr = n\frac{h}{2\pi} \quad thus\ v = \frac{nh}{2\pi mr} \ldots\ldots\ldots.(II)$$
$$Thus, v^2 = \frac{n^2h^2}{4\pi^2m^2r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(III)$$
Comparing (I) and (III) we get
$$\frac{e^2}{4\pi\varepsilon_o mr} = \frac{n^2h^2}{4\pi^2m^2r^2}$$
$$Thus, \quad r = \frac{\varepsilon_o n^2h^2}{\pi me^2}$$

This is the equation of the radius of n^{th} Bohr orbit
For n=1, $r_1 = \frac{\varepsilon_o h^2}{\pi me^2}$ = 0.537
In general $r_n = r_1 n^2$, thus r α n^2
The radius of Bohr orbit is directly proportional to the square of principle quantum number.
NOTE: speed of the electron can be got by substituting r in (II) we get
$$v = \frac{e^2}{2\varepsilon_o nh}, \quad thus\ v\ \alpha\ \frac{1}{n}$$

NOTE: angular velocity ω can be got using v = rω & freq f using ω=2πf
NOTE: Linear momentum = mv and angular momentum = $\frac{nh}{2\pi}$

ENERGY of electron in Bohr's orbit:
According to Bohr's postulate, electron revolves around in circular orbit. It has two energies; potential energy and kinetic energy. Therefore total energy possessed by an electron is given by
Total energy E = PE + KE
The electric potential V = $\frac{1}{4\pi\varepsilon_o}\frac{e}{r}$
The PE of an electron in the field of nucleus is (PE) = potential x charge
PE = $\frac{1}{4\pi\varepsilon_o}\frac{e}{r}(-e) = -\frac{e^2}{4\pi\varepsilon_o r}$

By Bohr's first postulate, $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} \qquad thus\ mv^2 = \frac{e^2}{4\pi\varepsilon_o r}$
Therefore, $KE = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_o r}$
Thus, E = PE + KE = $-\frac{e^2}{4\pi\varepsilon_o r} + \frac{e^2}{8\pi\varepsilon_o r} = -\frac{e^2}{8\pi\varepsilon_o r}$
$$Substituting\ r\ we\ get, \quad E = -\frac{me^4}{8\varepsilon_o{}^2n^2h^2}$$

Minus sign signifies that the electron is bound to the nucleus by the force of attraction and hence one has to do work on the electron to make it free from the atom.
$$E_n\ \alpha\ \frac{1}{n^2}$$

Energy of electron in the n^{th} Bohr orbit is inversely proportional to the square of principal quantum number.
For n=1 , E_1 = – 13.6 eV . Thus for the n^{th} Bohr orbit En = $\frac{-13.6}{n^2}eV$

Thus the valid energies of electrons in hydrogen atom are – 13.6 eV, $-\frac{13.6}{4}$ eV, $-\frac{13.6}{9}$ eV in the 1st , 2nd , and 3rd orbit respectively.
The Binding Energy of electron or ionization energy of hydrogen atom is minimum energy required to make it free from the nucleus. For electron in n^{th} orbit it is $\frac{me^4}{8\varepsilon_o{}^2n^2h^2}$ and 0 for n=∞

DERIVATION of Wave Number for Spectral Lines of Hydrogen

In a normal, unexcited hydrogen atom, an electron resides in innermost orbit (n=1). This is called ground state. The energy of an electron in the ground state is – 13.6 eV. If the atom is subjected to external energy equal or greater than 13.6eV, electron becomes free. This process is called ionization.

Suppose an electron jumps from n^{th} higher orbit to p^{th} lower orbit and E_n and E_p be their energies in their respective orbits, then

$$E_n = -\frac{me^4}{8{\varepsilon_o}^2n^2h^2} \quad and \quad E_p = -\frac{me^4}{8{\varepsilon_o}^2p^2h^2}$$

According to Bohr's 3rd postulate

Energy emitter = hv = $E_n - E_p$ = $-\frac{me^4}{8{\varepsilon_o}^2n^2h^2} + \frac{me^4}{8{\varepsilon_o}^2p^2h^2}$

$$hv = \frac{me^4}{8{\varepsilon_o}^2h^2}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)$$

$$Thus, v = \frac{me^4}{8{\varepsilon_o}^2h^3}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)$$

$$\frac{c}{\lambda} = \frac{me^4}{8{\varepsilon_o}^2h^3}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = \frac{me^4}{8{\varepsilon_o}^2h^3c}\left(\frac{1}{p^2}-\frac{1}{n^2}\right) = R\left(\frac{1}{p^2}-\frac{1}{n^2}\right)$$

$$where\ R = Rydberg's\ constant = 1.093\ X\ 10^7 m^{-1}$$

$$\frac{1}{\lambda}\ is\ called\ wave\ number\ also\ denoted\ as\ \bar{v}$$

HYDROGEN SPECTRUM / HYDROGEN SPECTRAL LINES / ENERGY LEVEL DIAGRAM OF HYDROGEN ATOM:

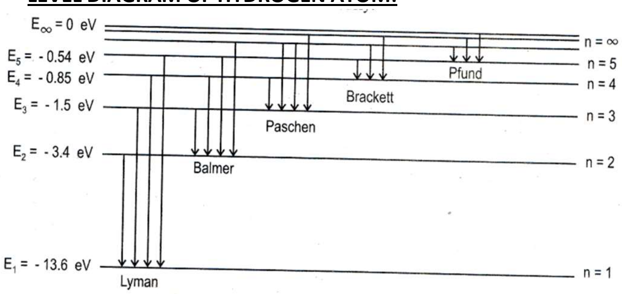


Fig 18.3: Energy level diagram for hydrogen atom

A series of spectral lines is radiated due to transition of electrons from different outer orbits to fixed inner orbit. The spectral series observed for hydrogen are:

Lyman Series: Due to transition of electrons from different outer orbits to first Bohr orbit (p=1)

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2}-\frac{1}{n^2}\right), where\ n = 2,3,4,5\ \ldots..$$

This series lies in the ultraviolet region of the spectrum.

Balmer Series: Due to transition of electrons from different outer orbits to second Bohr orbit (p=2)

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2}-\frac{1}{n^2}\right), where\ n = 3,4,5,6\ \ldots..$$

This series lies in the VISIBLE region of the spectrum.

The lines are named H_α, H_β, H_γ and H_δ

Paschen Series: Due to transition of electrons from different outer orbits to third Bohr orbit (p=3)

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2}-\frac{1}{n^2}\right), where\ n = 4,5,6,7\ \ldots..$$

This series lies in the infrared region of the spectrum.

Brackett Series: Due to transition of electrons from different outer orbits to fourth Bohr orbit (p=4)

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2}-\frac{1}{n^2}\right), where\ n = 5,6,7,8\ \ldots..$$

This series lies in the near – infrared region of the spectrum.

Pfund Series: Due to transition of electrons from different outer orbits to fifth Bohr orbit (p=5)

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2}-\frac{1}{n^2}\right), where\ n = 6,7,8,9\ \ldots..$$

This series lies in the far – infrared region of the spectrum.

NOTE: Series Limit means the smallest wavelength i.e. from n=∞

Lyman Series limit: $\frac{1}{\lambda} = R\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right) = \frac{R}{1}, thus\ \boldsymbol{\lambda_{min}} = \frac{\boldsymbol{1}}{\boldsymbol{R}}$

Balmer Series limit: $\frac{1}{\lambda} = R\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right) = \frac{R}{4}, thus\ \boldsymbol{\lambda_{min}} = \frac{\boldsymbol{4}}{\boldsymbol{R}}$

Paschen Series limit: $\frac{1}{\lambda} = R\left(\frac{1}{3^2}-\frac{1}{\infty^2}\right) = \frac{R}{9}, thus\ \boldsymbol{\lambda_{min}} = \frac{\boldsymbol{9}}{\boldsymbol{R}}$

Brackett Series limit: $\frac{1}{\lambda} = R\left(\frac{1}{4^2}-\frac{1}{\infty^2}\right) = \frac{R}{16}, thus\ \boldsymbol{\lambda_{min}} = \frac{\boldsymbol{16}}{\boldsymbol{R}}$

Pfund Series limit: $\frac{1}{\lambda} = R\left(\frac{1}{5^2}-\frac{1}{\infty^2}\right) = \frac{R}{25}, thus\ \boldsymbol{\lambda_{min}} = \frac{\boldsymbol{25}}{\boldsymbol{R}}$

De BROGLIE'S HYPOTHESIS (Matter Waves):

In 1924, Louis de Broglie suggested that if radiant energy has both wave and particle nature, then particle (matter) must have a wave associated with it.

The wavelength associated with a particle of mass m moving with a speed v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}, where\ h\ is\ planck's\ constant\ and\ p\ is\ momentum$$

This explain dual nature of matter. On left side, it has wavelength λ of wave and on right side the momentum p of particle and Planck's constant relates them.

As v→0, λ→∞ and as v→∞, λ→0

This implies that matter are associated with material particles only if they are moving. Greater the momentum of the particle, the shorter the wavelength. Matter waves travel faster than light. The velocity of matter waves is not constant because it depends upon the velocity of particle.

The de Broglie wavelength is independent of the charge of particle.

The intensity of wave at a point represents the probability of the associated particle being there.

Proof:

According to Planck's quantum theory energy of photon E = hv = $\frac{hc}{\lambda}$

According to Einstein's mass energy relation E = mc^2

Thus $mc^2 = \frac{hc}{\lambda}, \quad or \quad \lambda = \frac{h}{mc} = \frac{h}{p}$

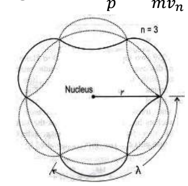
Verification of 2nd Postulate by de Broglie's:

For an electron moving in the n^{th} orbit or radius r_n, the distance traveled is $2\pi r_n$ and that should be an integral multiple of wavelength.

$2\pi r_n = n\lambda$, where n=1,2,3,4…… But by de Broglie's $\lambda = \frac{h}{p} = \frac{h}{mv_n}$

Thus, $2\pi r_n = n\frac{h}{mv_n}$ or $mv_nr_n = \frac{nh}{2\pi}$

This is the quantum condition proposed by Bohr for angular momentum of the electron in 2nd postulate.



Wavelength of an accelerated electron:

If an electron of mass m is accelerated by potential V, the work done on electron increases its KE

Work done = E = eV = ½ mv² = ½ m²v²/m = $\frac{p^2}{2m}$ .Thus $p = \sqrt{2mE} = \sqrt{2meV}$

substitute in de Broglie's equation $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}$ Å

This relation gives the wavelength of electron accelerated by p.d. 'V'.

(DASISSON AND GERMER EXPERIMENT REFER TEXTBOOK)